SCATTERING POLARIMETRY
PART 1

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(Slide courtesy Prof. E. Pottier and Prof. L. Ferro-Famil)
That's how it looks!

REAL ELECTRIC FIELD VECTOR

\[ \vec{E}(z,t) = \begin{cases} 
E_x &= E_{0x} \cos(\omega t - kz - \delta_x) \\
E_y &= E_{0y} \cos(\omega t - kz - \delta_y) \\
E_z &= 0 
\end{cases} \]
Wave Polarisation

- An electromagnetic (EM) plane wave has time-varying Electric and Magnetic Field components in a plane perpendicular to the direction of travel.

- The two fields are orthogonal to one another, and are described by Maxwell's equations.

- The fields propagate at the speed of light in "free space", which includes most realistic atmospheric conditions.

- Three parameters are necessary and sufficient to describe the propagation of EM waves in a given medium: dielectric constant (or permittivity), permeability and conductivity.
Wave Polarisation

- In general, when an EM wave is emitted from a source, such as a radar antenna, it propagates in all available directions, (with a specific field strength and phase in each direction).

- At a long distance from the antenna, we can assume that the wave front lies on a plane, rather than on the surface of a sphere.

- Since we are only interested in what happens to the wave along one specific direction, the "plane wave" assumption is appropriate.
Polarization is an important property of a plane EM wave.

Polarization refers to the alignment and regularity of the Electric and Magnetic Field components of the wave, in a plane perpendicular to the direction of propagation.

By convention, we direct our attention to the Electric Field component of the wave, as the orthogonal Magnetic Field component "follows" it according to Maxwell's equations (the Magnetic Field is directly related to the Electric Field, and can always be calculated from it).
Wave Polarisation

- The waveform of the Electric Field vector can be predictable or random, or a combination of both.

- A random component is like pure noise, with neither a recognizable frequency nor a pattern to its amplitude.

- An example of a predictable component is a monochromatic sine wave, with a constant, single frequency and a constant amplitude.

- An EM wave that has no random component is called fully polarized.
Wave Polarisation

THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE

\[
\left( \frac{E_x}{E_{0x}} \right)^2 - 2 \frac{E_x E_y}{E_{0x} E_{0y}} \cos(\delta) + \left( \frac{E_y}{E_{0y}} \right)^2 = \sin^2(\delta)
\]

With: \( \delta = \delta_y - \delta_x \)
Wave Polarisation

- The polarisation of a plane electro-magnetic wave describes the orientation of the electric field as a function of time.

- In the general case
  - The locus of the E-field in a plane perpendicular to the direction of propagation is an ellipse
  - With special cases for linear and circular polarisation.
Wave Polarisation

- Radar uses an antenna that is designed to transmit and receive EM waves of a specific polarization.

- Antennas come in many forms, including horns, waveguides, dipoles and patches. In each case, the electric and mechanical properties of the antenna are such that the transmitted wave is almost purely polarized with a specific design polarization.

- In a simple radar system, the same antenna is often configured so that it is matched to the same polarization on reception (when an EM wave is incident upon it)
Wave Polarisation

- Signals with components in two orthogonal or basis polarizations are needed to create a wave with an arbitrary polarization.

- The two most common basis polarizations are horizontal linear or H, and vertical linear or V.

- Circular polarizations are also in use for some applications, e.g. weather radars. Their basis components are denoted by R for Right Hand Circular and L for Left Hand Circular.
In more complex radar systems, the antenna may be designed to transmit and receive waves at more than one polarization.

On transmit, waves of different polarizations can be transmitted separately, using a switch to direct energy to the different parts of the antenna in sequence (e.g. the H and V parts).

In some cases the two parts can be used together, for example, a circular polarized signal can be transmitted by feeding the H and V parts of the antenna simultaneously, with signals of equal strength and a 90° phase difference.
Wave Polarisation

- Because the scatterer can change the polarization of the scattered wave to be different from the polarization of the incident wave, the radar antenna is often designed to receive the different polarization components of the EM wave simultaneously.

- For example, the H and V parts of an antenna can receive the two orthogonal components of the incoming wave, and the system electronics keep these two signals separate.
Wave Polarisation

- Denoting the transmit and receive polarizations by a pair of symbols, a radar system using H and V linear polarizations can thus have the following channels:
  - HH - for horizontal transmit and horizontal receive, (HH)
  - VV - for vertical transmit and vertical receive, (VV)
  - HV - for horizontal transmit and vertical receive (HV),
  - VH - for vertical transmit and horizontal receive (VH).
Wave Polarisation

- A radar system can have different levels of polarization complexity:
  - Single polarized - HH or VV or HV or VH
  - Dual polarized - HH and HV, VV and VH, or HH and VV
  - Four (Quad) polarizations - HH, VV, HV, and VH
Wave Polarisation

- The importance for having fully polarimetric signal !!!

- A quadrature (fully) polarized (i.e. polarimetric) radar uses these four polarizations, and measures the phase difference between the channels as well as the magnitudes.

- Some dual polarized radars also measure the phase difference between channels, as this phase plays an important role in polarimetric information extraction.
Wave Polarisation

\[
E = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{0x} e^{j\delta_x} \\ E_{0y} e^{j\delta_y} \end{bmatrix}
\]

\[
= A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j \sin(\tau) \\ j \sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x
\]

**Complex Polarisation Ratio**

\[
\rho(\hat{x},\hat{y}) = \frac{E_y}{E_x} = \frac{E_{0y}}{E_{0x}} e^{j(\delta_y - \delta_x)} = \frac{\tan(\phi) + j \tan(\tau)}{1 - j \tan(\phi) \tan(\tau)}
\]

Independent of amplitude and absolute phase.
Wave Polarisation

REAL REPRESENTATION OF THE POLARISATION STATE OF A MONOCHROMATIC WAVE

\[ E \cdot E^{T*} = \begin{bmatrix} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{bmatrix} \]

PAULI MATRICES GROUP

\[ \sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \]

\[ E \cdot E^{T*} = \frac{1}{2} \{ g_0 \sigma_0 + g_1 \sigma_1 + g_2 \sigma_2 + g_3 \sigma_3 \} = \frac{1}{2} \begin{bmatrix} g_0 + g_1 & g_2 - jg_3 \\ g_2 + jg_3 & g_0 - g_1 \end{bmatrix} \]

\{ g_0, g_1, g_2, g_3 \} STOKES PARAMETERS
Wave Polarisation

**JONES VECTOR**

\[ E = \begin{bmatrix} E_x = E_{ox} e^{i\delta_x} \\ E_y = E_{oy} e^{i\delta_y} \end{bmatrix} \]

**STOKES VECTOR**

\[ \mathbf{g}_E = \begin{bmatrix} g_0 = |E_x|^2 + |E_y|^2 \\ g_1 = |E_x|^2 - |E_y|^2 \\ g_2 = 2\Re(E_x E_y^*) \\ g_3 = -2\Im(E_x E_y^*) \end{bmatrix} \]

WAVE POLARISATION STATE ESTIMATION FROM INTENSITIES MEASUREMENTS
Wave Polarisation

STOKES VECTOR

\[
\mathbf{g}_E = \begin{bmatrix}
  g_0 &= E_{0x}^2 + E_{0y}^2 \\
  g_1 &= E_{0x}^2 - E_{0y}^2 \\
  g_2 &= 2E_{0x}E_{0y}\cos(\delta) \\
  g_3 &= 2E_{0x}E_{0y}\sin(\delta)
\end{bmatrix} = \begin{bmatrix}
  g_0 &= A^2 \\
  g_1 &= A^2 \cos 2\phi \cos 2\tau \\
  g_2 &= A^2 \sin 2\phi \cos 2\tau \\
  g_3 &= A^2 \sin 2\tau
\end{bmatrix}
\]

GEOMETRICAL PARAMETERS

ORIENTATION ANGLE

\[\tan 2\phi = 2 \frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2} \cos \delta = \frac{g_2}{g_1}\]

ELLipticity ANGLE

\[\sin 2\tau = 2 \frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2} \sin \delta = \frac{g_3}{g_0}\]
Wave Polarisation

**STOKES VECTOR**

\[
\mathbf{g}_E = \begin{bmatrix}
    g_0 \\
    g_1 \\
    g_2 \\
    g_3 \\
\end{bmatrix} = \begin{bmatrix}
    |E_x|^2 + |E_y|^2 \\
    |E_x|^2 - |E_y|^2 \\
    2\Re(E_x E_y^*) \\
    -2\Im(E_x E_y^*) \\
\end{bmatrix} = \begin{bmatrix}
    E_{0x}^2 + E_{0y}^2 \\
    E_{0x}^2 - E_{0y}^2 \\
    2E_{0x}E_{0y}\cos(\delta) \\
    2E_{0x}E_{0y}\sin(\delta) \\
\end{bmatrix} = \begin{bmatrix}
    A^2 \\
    A^2 \cos 2\phi \cos 2\tau \\
    A^2 \sin 2\phi \cos 2\tau \\
    A^2 \sin 2\tau \\
\end{bmatrix}
\]

\{g_0\}  \quad \text{TOTAL WAVE INTENSITY}

\{g_1, g_2, g_3\}  \quad \text{POLARISED WAVE INTENSITIES}

\[
g_0^2 = g_1^2 + g_2^2 + g_3^2  \quad \text{WAVE FULLY POLARISED}
\]

\{g_1, g_2, g_3\}  \quad \text{Spherical Coordinates of a point P on a sphere with radius } g_0
Wave Polarisation

- Deterministic Scattering
  - Completely Polarised Wave

- Random Scattering
  - Partially Polarised Wave

Polarisation Ellipse varies in time
Amplitude, Phase: Random processes

Statistical Description
Wave Polarisation

**JONES VECTORS** $\{E\}$

**WAVE COVARIANCE MATRIX**

$$\langle [J]\rangle = \langle EE^T* \rangle = \begin{bmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{bmatrix}$$

$$\langle [J]\rangle = \frac{1}{2} \begin{bmatrix} \langle g_0 \rangle + \langle g_1 \rangle & \langle g_2 \rangle - j\langle g_3 \rangle \\ \langle g_2 \rangle + j\langle g_3 \rangle & \langle g_0 \rangle - \langle g_1 \rangle \end{bmatrix}$$

$$\langle g_0 \rangle^2 \geq \langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2$$

**PARTIALLY POLARISED WAVES**
Wave Polarisation

WAVE COVARIANCE MATRIX

\[
\langle [J] \rangle = \langle E E^T \rangle = \begin{bmatrix}
\langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\
\langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle
\end{bmatrix}
\]

**DIAGONAL ELEMENTS**: INTENSITIES ON EACH OF THE 2 ORTHOGONAL COMPONENTS OF THE WAVE

**OFF-DIAGONAL ELEMENTS**: CROSS-CORRELATIONS BETWEEN THE 2 ORTHOGONAL COMPONENTS OF THE WAVE

\[
\text{Trace}(J) = \langle |E_x|^2 \rangle + \langle |E_y|^2 \rangle = A^2
\]

TOTAL WAVE INTENSITY

THE WAVE COVARIANCE MATRIX IS A 2x2 HERMITIAN POSITIVE SEMI-DEFINITE MATRIX
Wave Polarisation

- When a transmitted plane wave \( E' \) interacts with materials it may be reflected in different proportions horizontally and vertically and therefore change the polarisation properties of the wave.

- This scattering process can be described by means of a scattering matrix \( S \) transformation.

\[
E^t = \begin{bmatrix} E^t_h \\ E^t_v \end{bmatrix} e^{ik_0 r} \begin{pmatrix} E'_v \\ E'_h \end{pmatrix}
\]

Complex valued wave representation in V and H polarization
Wave Polarisation

The returned wave, transformed by the scattering process

\[
E^r = \frac{e^{ik_0r}}{r} SE^t
\]

\[
\begin{bmatrix}
E^r_h \\
E^r_v
\end{bmatrix} = \frac{e^{ik_0r}}{r} \begin{bmatrix}
S_{hh} & S_{hv} \\
S_{vh} & S_{vv}
\end{bmatrix} \begin{bmatrix}
E^t_h \\
E^t_v
\end{bmatrix}
\]
The elements of the scattering matrix, $[S]$, are known as the complex scattering amplitudes and describe how the scatterer transforms the polarisation of the incident wave.

Each scattering amplitude may be a function of:
- Frequency
- Illuminating angle
- Orientation of the scatterer relative to the co-ordinate system.
The theorem of reciprocity states that the two cross-polar terms are equal

\[ S_{vh} = S_{hv} \equiv S_x \]

For targets whose internal state is unaltered by the polarisation of the probing wave.
- Expected to be the case for most naturally occurring scatterers.
- Real data may not always obey the reciprocity theorem exactly due to statistical fluctuations and measurement errors.
- The cross-polar term, \( S_x \), is often taken as the average of \( S_{vh} \) and \( S_{hv} \) multiplied by \( \sqrt{2} \) to conserve the total power in the vector, and is performed to reduce the statistical non-equality found in real data.
Scattering Theory

- The mathematical representation of individual scattered waves and point targets.

- The signal measured by the radar system will be the resulting signal of many individual scattered waves over a distributed target area or volume.

- A natural target will not be a single pure scatterer and the target area may have a significant surface roughness or volume scattering component.
Scattering Theory

- The measured scattering matrix will therefore represent a statistical property of the target location.

- We generally consider the case of incoherent scattering, which is usually the case for the natural environment.

- Incoherent scattering assumes that the signal for a given target location is comprised of returns from an unknown number of individual waves.
Scattering Theory

- The magnitude and phase are determined by the many individual point conditions
  - Distance to target,
  - Surface angle
  - Material type
  - Polarising orientation.

- The vector sum of all such waves is the resulting signal measured by the antenna, and can be modelled as a two dimensional random walk in the basis.
It is assumed that the target area is sufficiently large and textured, relative to the illuminating wavelength.

So that the individual returns can be considered independent and the phase of the vector sum can be considered *uniformly random*.

The central limit theorem implies that when the number of individual scattering points per resolution cell is very large and the scattering medium is homogeneous.

The scattering process would be *Gaussian distributed*. 
If the scattering medium is not spatially homogeneous, or the resolution cell is not sufficiently large that the central limit theorem applies

- The distribution may be \textit{non-Gaussian}

In practice, the amplitude will be a measure of the average target reflectance
Scattering Theory

- The random phase means that this value is evenly shared between the real and imaginary returned signal.
  - i.e., the magnitude and phase will have equal average intensity, and
  - the individual values will be uncorrelated.
- Additionally, being magnitude and phase of the complex vector-field signal, implies that they should both measure plus or minus values centered around a mean of zero.
Scattering Theory

\[ E[x_i] = E[y_i] = 0 \]

\[ E[x_iy_i] = 0 \]

\[ E[x_i x_k] = E[y_i x_k] \]

\[ E[y_i x_k] = -E[x_i y_k] \]
WAVE POLARIMETRY

TRANSMITTER: X
RECEIVERS: X & Y

JONES VECTORS

\[ E_s = \begin{bmatrix} S_{XX} \\ S_{YY} \end{bmatrix} \]
SCATTERING POLARIMETRY

TRANSMITTER: X & Y
RECEIVERS: X & Y

SINCLAIR MATRICES

\[
[S] = \begin{bmatrix}
S_{XX} & S_{XY} \\
S_{YX} & S_{YY}
\end{bmatrix}
\]