SPECKLE FILTERING / SPECKLE STATISTICS

Dr. A. Bhattacharyya
(Slide courtesy Prof. E. Pottier and Prof. L. Ferro-Famil)
OBSERVATION POINT

SURFACE ROUGHNESS WAVELENGTH

SCATTERING FROM DISTRIBUTED SCATTERERS

COHERENT INTERFERENCES OF WAVES SCATTERED FROM MANY RANDOMLY DISTRIBUTED ELEMENTARY SCATTERERS INSIDE THE RESOLUTION CELL

GRANULAR NOISE

SPECKLE PHENOMENON
SPECKLE PHENOMENON

DISTORTION OF THE INTERPRETATION

SPECKLE FILTERING

HOMOGENEOUS AREA

SPECKLE REDUCTION (RADIOMETRIC RESOLUTION)

HETEROGENEOUS AREA

DETAILS PRESERVATION (SPATIAL RESOLUTION)
SPECKLE REDUCTION

MULTI-LOOK SAR PROCESSING (BoxCar)

Averaging Amplitude / Intensity (Not complex images) of neighboring pixels

Good Noise Smoothing

Spatial Resolution Loss - blurring edges - erasing thin lines
Loss of linear or point features ...

Why not? What happens?
SPECKLE : MULTIPlicative NOISE MODEL

« SPECKLE is a scattering phenomenon and not a noise. However, from the image SAR processing point of vue, the speckle can be modeled as multiplicative noise for extended target » (Lee, IGARSS-98)

$$\mathbf{y} = \begin{bmatrix} y_{HH} \\ y_{HV} \\ y_{VV} \end{bmatrix} = \begin{bmatrix} n_{HH} & 0 & 0 \\ 0 & n_{HV} & 0 \\ 0 & 0 & n_{VV} \end{bmatrix} \begin{bmatrix} x_{HH} \\ x_{HV} \\ x_{VV} \end{bmatrix} = \begin{bmatrix} x_{HH} n_{HH} \\ x_{HV} n_{HV} \\ x_{VV} n_{VV} \end{bmatrix}$$

**SCATTERING FIELD**  **NOISE**  **REFLECTIVITY DENSITY**

$$Y_{pqpq} = y_{pq} y_{pq}^* = X_{pqpq} v_{pqpq}$$

**INTENSITY**

$$A_{pqpq} = \sqrt{Y_{pqpq}} = \sqrt{y_{pq} y_{pq}^*}$$

**AMPLITUDE**
### Homogeneous Area: N Look Case

#### Intensity

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{pqpq}$</td>
<td>$y_{pq} y_{pq}^*$</td>
</tr>
<tr>
<td>$Y$</td>
<td>$\frac{1}{N} \sum_i Y_i$</td>
</tr>
<tr>
<td>$P_{NY}(Y/X_0)$</td>
<td>$\frac{N^N Y^{N-1}}{\Gamma(N) X_0^N} e^{\frac{NY}{X_0}}$</td>
</tr>
<tr>
<td>$E(Y)$</td>
<td>$X_0$</td>
</tr>
<tr>
<td>$\text{var}(Y)$</td>
<td>$\frac{X_0^2}{N}$</td>
</tr>
<tr>
<td>$CV_{NY}$</td>
<td>$\frac{1}{\sqrt{N}}$</td>
</tr>
</tbody>
</table>

#### Amplitude

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{pqpq}$</td>
<td>$\sqrt{y_{pq} y_{pq}^*}$</td>
</tr>
<tr>
<td>$A$</td>
<td>$\sqrt{\frac{1}{N} \sum_i Y_i}$</td>
</tr>
<tr>
<td>$P_{NA}(A/X_0)$</td>
<td>$\frac{2N^N A^{2N-1}}{\Gamma(N) X_0^N} e^{\frac{NA^2}{X_0}}$</td>
</tr>
<tr>
<td>$E(A)$</td>
<td>$\sqrt{\frac{X_0 \Gamma(N+\frac{1}{2})}{N \Gamma(N)}}$</td>
</tr>
<tr>
<td>$\text{var}(A)$</td>
<td>$\frac{X_0}{N} \left( N - \frac{\Gamma^2(N+\frac{1}{2})}{\Gamma^2(N)} \right)$</td>
</tr>
<tr>
<td>$CV_{NA}$</td>
<td>$\sqrt{\frac{N \Gamma^2(N)}{\Gamma^2(N+\frac{1}{2})} - 1} \approx 0.523$</td>
</tr>
</tbody>
</table>
VECTORIAL SPECKLE FILTER

\[ [C] = yy^T \rightarrow \text{FILTER} \rightarrow [\hat{C}] = \hat{x}\hat{x}^T \]

COVARIANCE MATRIX


- POLARIMETRIC SPECKLE FILTERING SHOULD FILTER ALL ELEMENTS OF THE COVARIANCE MATRIX
- AVOIDING CROSS-TALK BETWEEN CHANNELS DUE TO THE FILTERING PROCESS
- PRESERVING POLARIMETRIC INFORMATION AND THE STATISTICAL CORRELATION BETWEEN THE CHANNELS
- PRESERVING SPATIAL RESOLUTION, FEATURES, EDGE SHARPNESS AND POINT TARGETS
Speckle appearing in synthetic aperture radar (SAR) images is due to the coherent interference of waves reflected from many elementary scatterers.

This effect causes a pixel-to-pixel variation in intensities, and the variation manifests itself as a granular noise pattern in SAR images.

Understanding SAR speckle statistics is essential for better information extraction by designing intelligent algorithms for speckle filtering, geophysical parameter estimation, and land-use, ground cover classification, etc.
When radar illuminates a surface that is rough on the scale of the radar wavelength, the returned signal consists of waves reflected from many elementary scatterers (or facets) within a resolution cell.

The distances between the elementary scatterers and the radar receiver vary due to the random location of scatterers.

Therefore, the distance from the scatterers to the radar is random.

The received waves from each scatterer, although coherent in frequency, are no longer coherent in phase.

A strong signal is received, if wavelets add relatively constructively;

A weak signal, if the waves are out of phase.
\[ \sum_{i=1}^{M} (x_i + jy_i) = \sum_{i=1}^{M} x_i + j \sum_{i=1}^{M} y_i = x + jy \]
A SAR image is formed by coherently processing returns from successive pulses.

This effect causes a pixel-to-pixel variation in intensity, and the variation manifests itself as a granular pattern, called speckle.

This pixel-to-pixel intensity variation in SAR image has a number of consequences.

The most obvious one is that the use of a single pixel intensity value as a measure of distributed targets’ reflectivity would be erroneous.
RAYLEIGH SPECKLE MODEL

• Under the conditions that
  (1) a large number of scatterers in a resolution cell of a homogeneous medium,

(2) the range distance is much larger than many radar wavelengths, and

(3) the surface is much rougher on the scale of the radar wavelength,

\[ \sum_{i=1}^{M} (x_i + jy_i) = \sum_{i=1}^{M} x_i + j \sum_{i=1}^{M} y_i = x + jy \]

• Assumed to have its phase uniformly distributed in the interval of \((-\pi, \pi)\).
• *Speckle possessed this property is called “fully developed speckle.”*
By the Central Limit theory

- The vector sum’s real and imaginary components, x and y, are independently and identically Gaussian (Normal) distributed with zero mean and a variance denoted as $s^2=2$.

- The amplitude $A$ defined as

$$A = \sqrt{x^2 + y^2}$$

has a Rayleigh probability distribution

$$p_1(A) = \frac{2A}{\sigma^2} \exp\left(-\frac{A^2}{\sigma^2}\right), \quad A \geq 0$$
The amplitude has a Rayleigh distribution with its mean

\[ M_1(A) = \sigma \sqrt{\pi}/2 \]

The variance

\[ \text{Var}_1(A) = (4 - \pi)\sigma^2/4 \]

The subscript “1” indicates that the speckle statistics are for the single-look SAR data. It is interesting to note that the ratio of the standard deviation to mean is independent of \( \sigma \)

The ratio equals to

\[ \sqrt{4/\pi - 1} = 0.5227. \]
This constant ratio is the basic characteristic of multiplicative noise

The intensity I defined as $I = x^2 + y^2 = A^2$ can be easily proved to have a negative exponential distribution

The mean

$$M_1(I) = \sigma^2$$

The variance

$$\text{Var}_1(I) = \sigma^4$$

The standard deviation to mean ratio is 1, which indicates that the speckle noise would appear more pronounced in intensity images than in the amplitude image, which has the ratio of 0.5227
Fully developed speckle

Partially developed speckle
A. 1-Look Intensity

B. Histogram of A (Exponential distribution)

C. 1-Look Amplitude

D. Histogram of C (Rayleigh distribution)

E. 4-Look Amplitude

F. Histogram of E (Chi distribution)
SPECKLE STATISTICS FOR MULTILOOK PROCESSED SAR IMAGES

- A common approach to speckle reduction is to average several independent estimates of reflectivity.

- In early SAR processing, this is accomplished by dividing the synthetic aperture length (or equivalently, azimuthal Doppler frequency spectrum) into N segments which are also known as looks.

- Each segment is processed independently to form either an intensity or an amplitude SAR image, and the N images are summed together to form an N-look SAR image.
The averaging process resembles the average of N independent samples, if samples are assumed statistically independent.

The N-look processing reduces the standard deviation of speckle by a factor of $\sqrt{N}$.

However, this is accomplished at the expense of azimuth resolution which is degraded by a factor of N.

In current SAR systems, the data are available either in multilook or in single-look complex format.
In general, the single-look complex data have a higher resolution in the azimuth direction than in the range direction.

For these data, multilook processing is accomplished by averaging neighboring single-look processed pixels in the azimuth direction to make the multilook pixel nearly square in pixel spacing.

It should be noted that if the summation were performed on the complex images rather than on amplitudes or intensities, no speckle reduction is achieved, because the process is identical to the vector sum of the total number of elementary scatterers from the N images. The statistics remain identical to that of 1-look SAR data.
For N-look intensity SAR images

\[ I_N = \frac{1}{N} \sum_{i=1}^{N} I_1(i) = \frac{1}{N} \sum_{i=1}^{N} (x(i)^2 + y(i)^2) \]

where \( x(i) \) and \( y(i) \) are the real and imaginary parts of the \( i \)th look (or sample).

Since \( x(i) \) and \( y(i) \) are independently Gaussian distributed,

it is well known that \( NI_N \) has a Chi-square distribution with \( 2N \) degrees of freedom.

Therefore, the PDF of the N-look intensity is described by

\[ p_N(I) = \frac{N^N I^{N-1}}{(N - 1)! \sigma^{2N}} \exp\left(-NI/\sigma^2\right), \quad I \geq 0 \]
The mean

\[ M_N(I) = \sigma^2 \]

The variance

\[ \text{Var}_N(I) = \sigma^4 / N, \]

The standard deviation to the mean ratio is reduced by the factor \( \frac{1}{\sqrt{N}} \) of the single look data.
The “multiplicative” nature of the speckle noise has been verified by scatter plots of sample standard deviation versus sample mean produced in many homogeneous areas in a SAR image.

The multiplicative nature of the speckle phenomenon manifests itself by the close fit of straight lines passing through the origin.

The slopes of the lines for the 1-look and 4-look amplitude SAR images are 0.54 and 0.26, respectively, which are reasonably close to the theoretical values of 0.5227 and 0.261.
TEXTURE MODEL

- The Rayleigh speckle model agrees reasonably well for measurements over homogeneous regions in SAR images with a coarse spatial resolution.

- But often fails over heterogeneous backscattering media by SAR with a finer resolution.

- For this reason many other statistical distributions, such as, the K-distribution, the log-normal, and Weibull distributions have been found to be useful in modeling the amplitude statistics.

- The K-distribution has its particular attractiveness, because it is derived based on a physical scattering process.

- And because the K-distribution reduces to the Rayleigh distribution in the case of homogeneous media.
Non-Gaussian Model

- It has been shown experimentally that the Gaussian model, with its bundle of equivalent distributions for different data formats, presents a good fit to real radar image data.
  
  - *when the scene is homogeneous, with low to moderate roughness and a high number of scatterers*

- Nevertheless, there is abundant empirical evidence that real data deviate from the model too.
  - Especially for images of urban environment, but also for natural terrain, such as rough sea and forest in general.
This is generally explained by the notion of texture thought of as variations in the mean radar reflectivity between pixels with the same thematic content, which is not accounted for in the Gaussian model.

The Gaussian model only encompasses statistical variation attributed to speckle.

Several distributions have been proposed for single polarisation amplitude and intensity, that imply non-Gaussian statistics for the scattering coefficient.

The Weibull distribution and the Log-normal distribution are two of the most popular examples.
- Even though neither of them bear links to physical modelling of the scattering process

- They have been shown to provide reasonably good fit to real data covering rough surfaces.