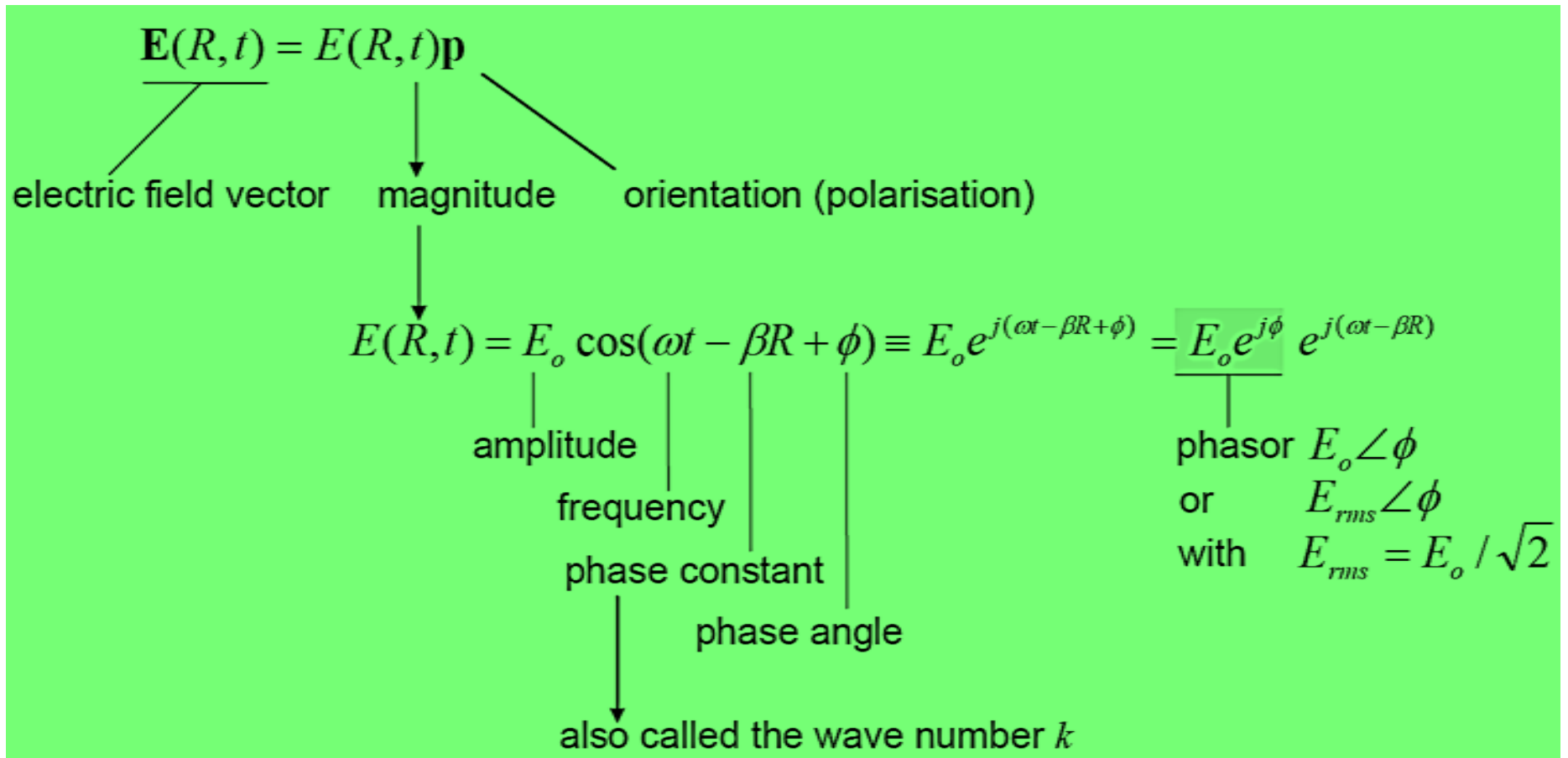


# THE ELECTROMAGNETIC FIELD THEORY II WAVE POLARIZATION

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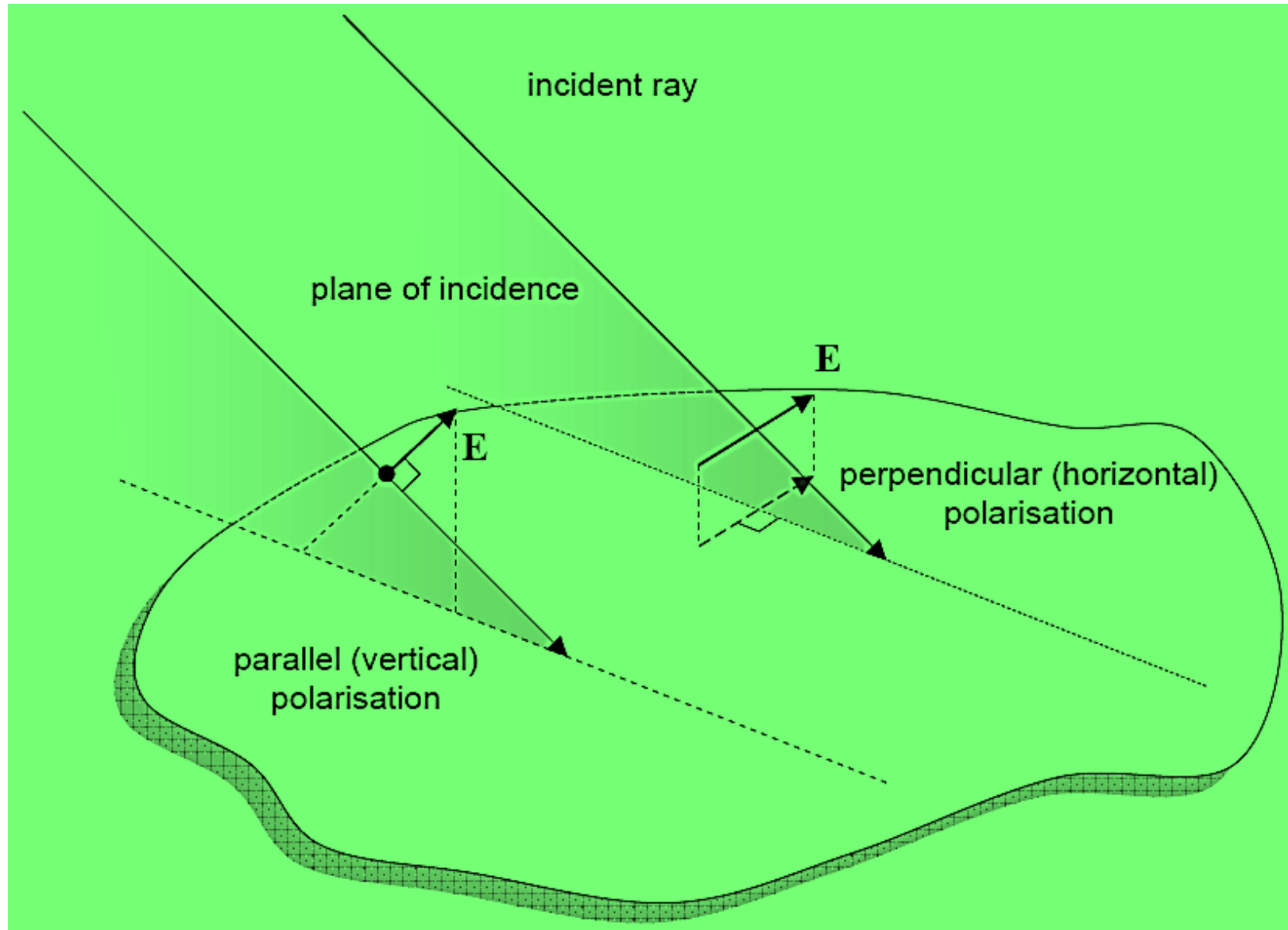
# Polarization



# Polarization

- The orientation of the Electric and the Magnetic field vectors define the polarization of the propagating wave
- The polarization of the propagating wave are not strictly important in terms of the propagation of radiation in free space
- When radiation strikes the ground the response of the surface material can be different for different orientation of the vectors
  - Need a convention to describe the direction in which the field vectors point

# Polarization

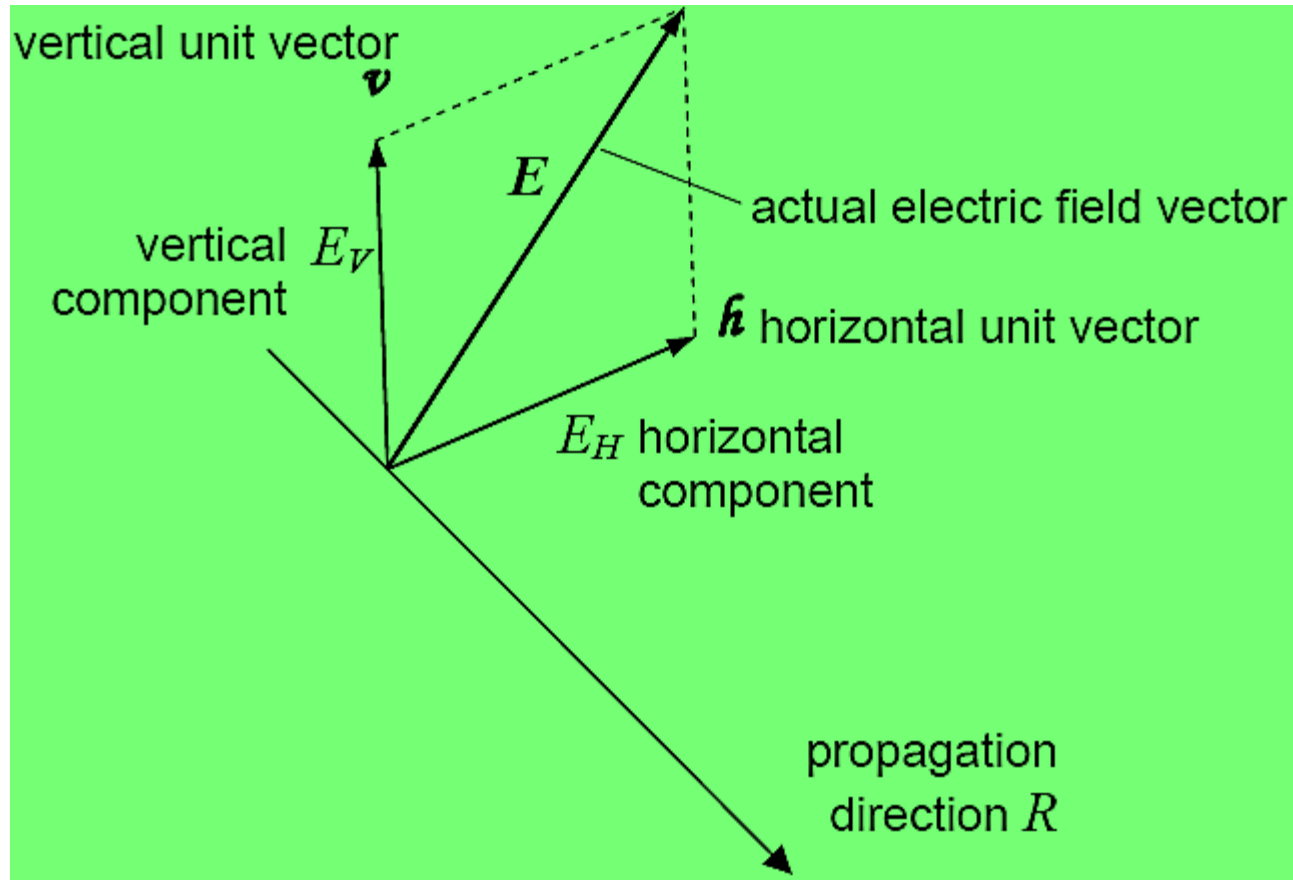


# Polarization

- Note that the *perpendicular* polarized wave is *horizontal* to the Earth's surface
  - In RS it is therefore more often called *horizontal polarization*
- *Not strictly correct*, parallel polarization is similarly referred to as *vertical polarization*
- The plane of polarization is that in which the Electric field vector oscillates sinusoidally

$$\mathbf{E} = E_H \mathbf{h} + E_V \mathbf{v}$$

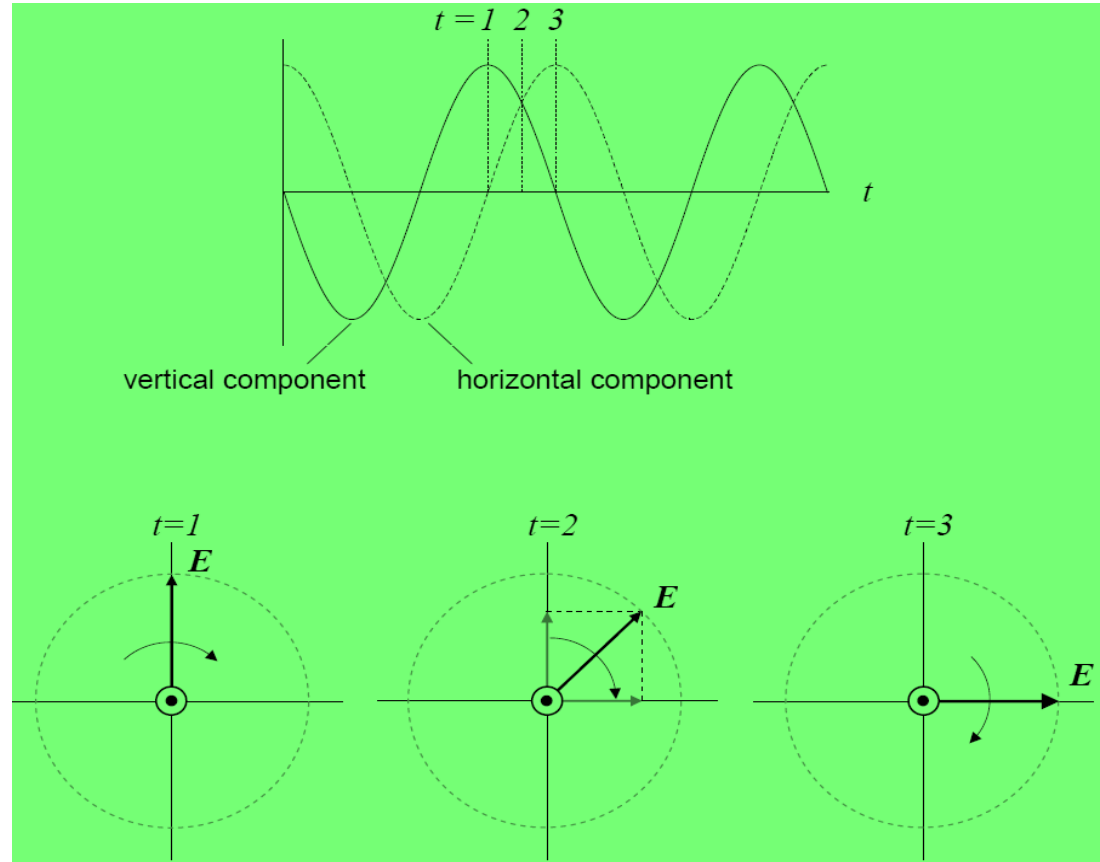
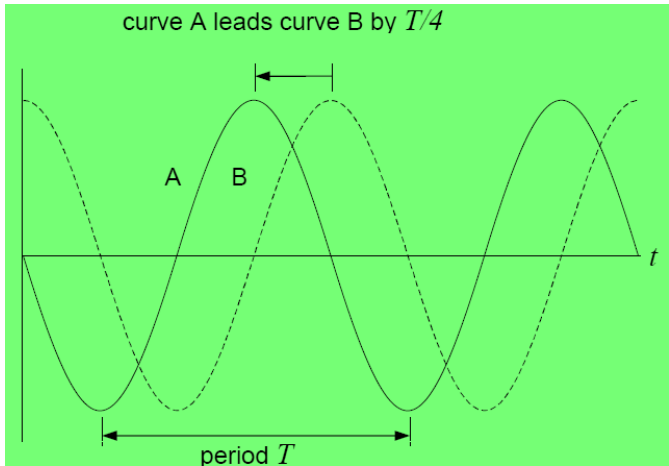
# Polarization



$$E_H = a_H \cos(\omega t - \beta R)$$
$$E_V = a_V \cos(\omega t - \beta R + \delta)$$

$\delta \Rightarrow$  **Phase difference**

# Polarization

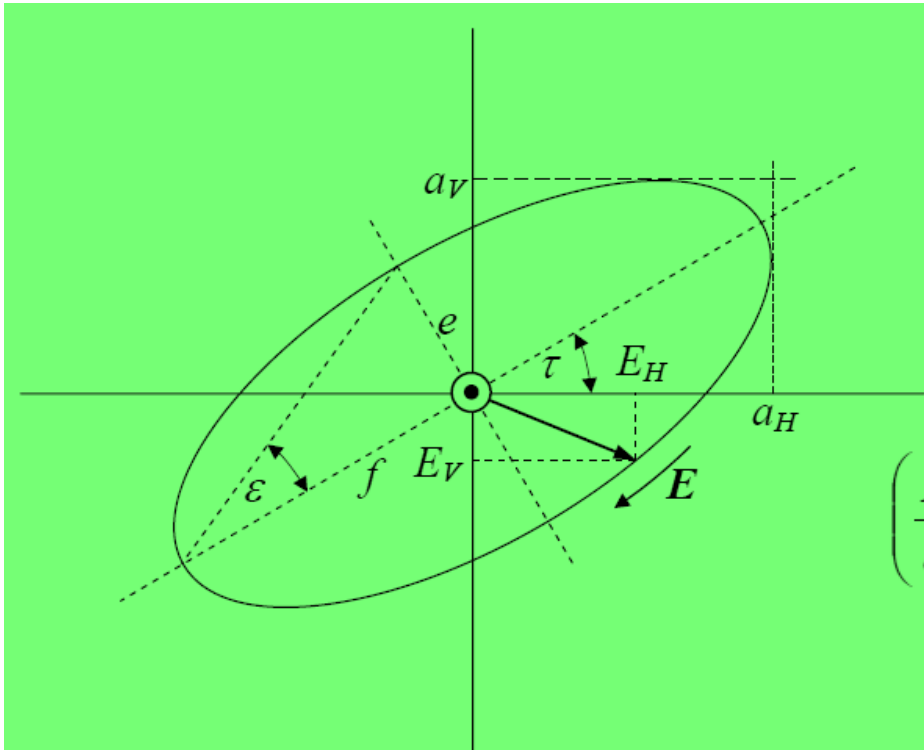


- The plot shows two components of the Electric field as functions of time at a given position in space to illustrate the significance of the phase difference

# Polarization

- Pure circular polarization only occurs when the two components have the same amplitude and the phase difference between them is  $90^\circ$
- In the most general case the approaching wave would be an ellipse
- There will be *left-elliptical* polarization and *right-elliptical* polarization depending on the sign of the phase angle between the components
- Circular (phase=  $90^\circ$ ) and linear polarization (phase=  $0^\circ$ ) are special cases. Their relative amplitudes will determine the orientation of the actual field vector

# Polarization



$$\left(\frac{E_V}{a_V}\right)^2 + \left(\frac{E_H}{a_H}\right)^2 - 2\frac{E_H E_V}{a_H a_V} \cos \delta = \sin^2 \delta$$

$$\delta = 0$$

$$\left(\frac{E_V}{a_V}\right)^2 + \left(\frac{E_H}{a_H}\right)^2 - 2\frac{E_H E_V}{a_H a_V} \cos \delta = \left(\frac{E_V}{a_V} - \frac{E_H}{a_H}\right)^2 = 0$$

$$\text{Slope} = a_v / a_h$$

$$\delta = \pi$$

$$\left(\frac{E_V}{a_V} + \frac{E_H}{a_H}\right)^2 = 0$$

$$\text{Slope} = -a_v / a_h$$

# Polarization

- Two properties of the ellipse relate directly to the polarization state of the radiation
  - Ellipticity or Eccentricity ( $\epsilon$ ) which describes how different it is from a circle or a straight line
  - Tilt ( $\tau$ ) or inclination with respect to the horizontal

$$\alpha = \tan^{-1} \frac{a_V}{a_H}$$

$$\tan 2\tau = \tan 2\alpha \cos \delta$$

$$\sin 2\epsilon = \sin 2\alpha \sin \delta$$

- This shows the explicit relationships between properties of the **wave** ( $a_h, a_v, \delta$ ) and those of the **polarization ellipse** ( $\tau, \epsilon$ )

# Polarization (Jones Vector)

- Another way of expressing the Electric field is by using the parameters of polarization ellipse

$$\mathbf{E} = a_H \exp j(\omega t - \beta R) \mathbf{h} + a_V \exp j(\omega t - \beta R + \delta) \mathbf{v} = \{a_H \mathbf{h} + a_V e^{j\delta} \mathbf{v}\} \exp j(\omega t - \beta R)$$

$$\mathbf{E} = \sqrt{a_H^2 + a_V^2} \{\cos \alpha \mathbf{h} + \sin \alpha \mathbf{v} e^{j\delta}\} \exp j(\omega t - \beta R)$$

$$\alpha = \tan^{-1} \frac{a_V}{a_H}$$

$$\mathbf{E} = A e^{\zeta} \begin{bmatrix} \cos \alpha \\ \sin \alpha e^{j\delta} \end{bmatrix} = A e^{\zeta} \mathbf{E}_J$$

***A → Amplitude***

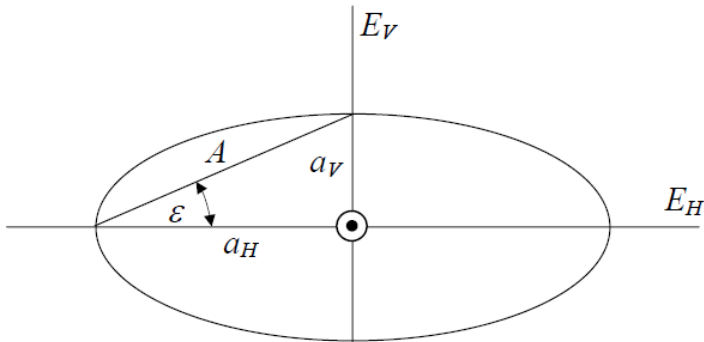
***ζ → Absolute phase***

***corresponding to  $-j\beta R$  resulting from propagation path***

***E<sub>J</sub> → Jones vector***

***The exponential factor in time is dropped since it applies to all fields***

# Polarization (Jones Vector)



$$\alpha = \varepsilon \Rightarrow \delta = \pm 90^\circ \text{ if } \alpha \neq 0$$

$$\Rightarrow e^{j\delta} = \pm j$$

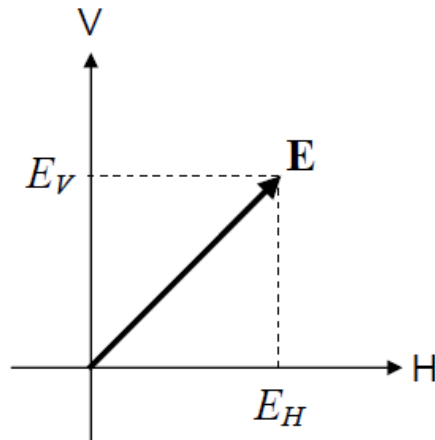


$$\mathbf{E} = Ae^{\zeta} \begin{bmatrix} \cos \varepsilon \\ \pm j \sin \varepsilon \end{bmatrix} = Ae^{\zeta} \mathbf{E}_J$$

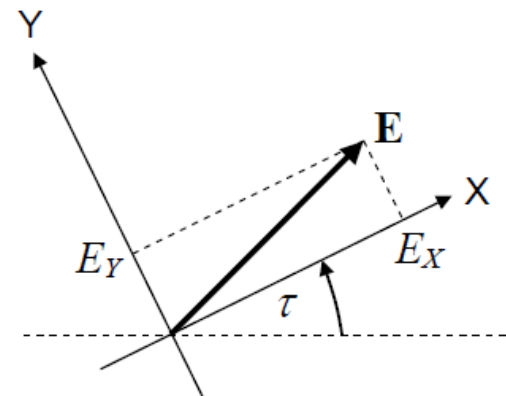
**We can absorb the sign into eccentricity  $\rightarrow +j \sin \varepsilon$  correspond to left elliptical polarization**

# Polarization (Jones Vector)

- We can transform the vector so that it applies to more general case of the inclined ellipse by rotating the axes clockwise by the inclination angle



$$\begin{bmatrix} E_X \\ E_Y \end{bmatrix} = \begin{bmatrix} \cos \tau & \sin \tau \\ -\sin \tau & \cos \tau \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix} = \mathbf{T}(\tau) \begin{bmatrix} E_H \\ E_V \end{bmatrix}$$



$$\mathbf{T}^{-1}(\tau) = \begin{bmatrix} \cos \tau & -\sin \tau \\ \sin \tau & \cos \tau \end{bmatrix}$$

$$\mathbf{E} = A e^{j\zeta} \begin{bmatrix} \cos \tau & -\sin \tau \\ \sin \tau & \cos \tau \end{bmatrix} \begin{bmatrix} \cos \varepsilon \\ j \sin \varepsilon \end{bmatrix}$$



**The Electric field vector in the most general case**

# Polarization (Jones Vector)

horizontal polarisation

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

vertical polarisation

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

right circular polarisation

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

left circular polarisation

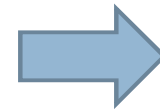
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$

+45° linear polarisation

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

-45° linear polarisation

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



**Some common  
Jones vectors**

# Polarization (Jones Vector)

- Circular polarization as a basis vector system
- We have considered a travelling wave as a combination of horizontal and vertical components
  - It is possible to choose *right circularly polarized* and *left circularly polarized* fields components as the basis

$$\mathbf{E} = E_L \boldsymbol{\ell} + E_R \mathbf{r} \equiv E_H \mathbf{h} + E_V \mathbf{v}$$

- The unit vectors  $\boldsymbol{\ell}$  and  $\mathbf{r}$  are unit vectors rotate around the unit circle in their respective directions carrying the relevant field magnitude with them

# Polarization (Jones Vector)

- Purely left circular polarization wave will have  $E_R = 0$ 
  - The horizontal and vertical field have the same amplitude and the vertical component leads  
 has a positive phase angle of  $90^\circ$

$$E_L \mathbf{l} = \exp j(\omega t - \beta R) \mathbf{h} + \exp j(\omega t - \beta R + \pi / 2) \mathbf{v}$$

- The magnitudes are assumed to be unity and  $E_L$  will have the same dependence on time and position as its two components

 which can be removed as a common factor  
 $\exp j(\omega t - \beta R)$

$$|E_L| \mathbf{l} = \mathbf{h} + e^{j\pi/2} \mathbf{v} = \mathbf{h} + j \mathbf{v}$$

Since  $\mathbf{l}$ ,  $\mathbf{h}$  and  $\mathbf{v}$  are unit vectors   $|E_L| = \sqrt{2}$

## Polarization (Jones Vector)

$$\boldsymbol{\ell} = \frac{1}{\sqrt{2}}(\boldsymbol{h} + e^{j\pi/2}\boldsymbol{v}) \equiv \frac{1}{\sqrt{2}}(\boldsymbol{h} + j\boldsymbol{v})$$

- In a similar manner a right circularly polarized wave will have its unit vector

$$\boldsymbol{r} = \frac{1}{\sqrt{2}}(\boldsymbol{h} - j\boldsymbol{v})$$

$$\begin{aligned}\boldsymbol{E} &= E_L \frac{1}{\sqrt{2}}(\boldsymbol{h} + j\boldsymbol{v}) + E_R \frac{1}{\sqrt{2}}(\boldsymbol{h} - j\boldsymbol{v}) \\ &= \frac{1}{\sqrt{2}}(E_L + E_R)\boldsymbol{h} + \frac{j}{\sqrt{2}}(E_L - E_R)\boldsymbol{v}\end{aligned}$$

$$\begin{aligned}E_H &= \frac{1}{\sqrt{2}}(E_L + E_R) \\ E_V &= \frac{j}{\sqrt{2}}(E_L - E_R)\end{aligned}$$

# Polarization (Jones Vector)

- In matrix form :

$$\begin{bmatrix} E_H \\ E_V \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \begin{bmatrix} E_L \\ E_R \end{bmatrix}$$

- This indicates how the *linear fields* can be computed from the *circular components*
- The circular components in terms of the linear components

$$\begin{bmatrix} E_L \\ E_R \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix}$$

$$\mathbf{h} = \frac{1}{\sqrt{2}}(\mathbf{r} + \mathbf{l})$$

$$\mathbf{v} = \frac{j}{\sqrt{2}}(\mathbf{r} - \mathbf{l})$$

## Polarization (Jones Vector)

$$\mathbf{h} = \frac{1}{\sqrt{2}}(\mathbf{r} + \mathbf{l}) \quad \mathbf{v} = \frac{j}{\sqrt{2}}(\mathbf{r} - \mathbf{l})$$

- A **horizontal polarized** wave is made up of **right** and **left** circularly polarized waves starting in phase (and contra-rotating)
- A **vertical polarized** wave is made up of the two contra-rotating starting in anti-phase
- The  $j$  is a time phase term common to both components advancing them by  $90^\circ$

# Polarization (Stokes Vector)

- Stokes parameters provide a very convenient means to describe the power density relationships in an EM wave in radar
- For a single frequency signal (monochromatic) Stokes parameters are defined as

$$s_0 = a_H^2 + a_V^2$$

$$s_1 = a_H^2 - a_V^2$$

$$s_2 = 2a_H a_V \cos \delta$$

$$s_3 = 2a_H a_V \sin \delta$$

$$s_0^2 = s_1^2 + s_2^2 + s_3^2$$

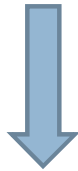
## Polarization (Stokes Vector)

- $S_0 \rightarrow$  *The amplitude squared or intensity of the actual field vector. It is directly proportional to the power density being carried by the wave*
- $S_1 \rightarrow$  *This indicates whether the wave is more horizontally than vertically polarized*
- $S_2$  &  $S_3 \rightarrow$  *Indicate the ellipticity of the wave's polarization*
  - *If  $\delta = 0$  we have linear polarization and  $S_3=0$*
  - *If  $\delta = 90^\circ$   $S_2=0$  and the polarization ellipse will aligned vertically or horizontally*
  - *It will be circular if the magnitudes of the vertical and horizontal components are also equal*

# Polarization (Stokes Vector)

$$\mathbf{s} = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} a_H^2 + a_V^2 \\ a_H^2 - a_V^2 \\ 2a_H a_V \cos \delta \\ 2a_H a_V \sin \delta \end{bmatrix} = \begin{bmatrix} |E_H|^2 + |E_V|^2 \\ |E_H|^2 - |E_V|^2 \\ 2\operatorname{Re}(E_H^* E_V) \\ 2\operatorname{Im}(E_H^* E_V) \end{bmatrix} = I_0 \begin{bmatrix} 1 \\ \cos 2\tau \cos 2\varepsilon \\ \sin 2\tau \cos 2\varepsilon \\ \sin 2\varepsilon \end{bmatrix}$$

$$I_0 = s_0$$



***Relative phase  
representation***



***Complex phasor  
representation***



***Representation in  
terms of principle  
angles of the  
polarization ellipse***

# Polarization (Stokes Vector)

- The Stokes vector can be represented in different forms represented as:

$$\mathbf{s} = \mathbf{R}\mathbf{g}$$

$$\mathbf{R} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -j & j \end{bmatrix}$$

$$\mathbf{g} = \begin{bmatrix} E_H E_H^* \\ E_V E_V^* \\ E_H E_V^* \\ E_V E_H^* \end{bmatrix} = \begin{bmatrix} |E_H|^2 \\ |E_V|^2 \\ E_H E_V^* \\ E_V E_H^* \end{bmatrix}$$

$$\mathbf{R}_c = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}$$

$$\mathbf{g}_c = \mathbf{E} \otimes \mathbf{E}^* = \begin{bmatrix} E_H E_H^* \\ E_H E_V^* \\ E_V E_H^* \\ E_V E_V^* \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} E_H \\ E_V \end{bmatrix}$$

$\mathbf{g}_c \rightarrow$  Coherency vector

# Polarization (Stokes Vector) Question ?

- Is the absolute phase ( $\delta_x$ ) or ( $\delta_y$ ) preserved in Stokes vector representation of waves ?

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- In Synthetic Aperture Radar (SAR) remote sensing the wave is transmitted in a narrowband
- The transmitted and received waves are narrowband about the central frequency  $\bar{\omega}$

$$\bar{\omega} - \frac{1}{2}\Delta\omega \leq \omega \leq \bar{\omega} + \frac{1}{2}\Delta\omega \quad \text{with } \frac{\Delta\omega}{\bar{\omega}} \ll 1$$

- The wave may still be interpreted as a plane wave is said to be quasi-monochromatic

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- The components  $E_1$  and  $E_2$  of the real vector  $\Re(\bar{E})$  is

$$E_x(z,t) = a_x(t) \cos(\bar{\omega}t - \bar{k}z + \delta_x(t))$$

$$E_y(z,t) = a_y(t) \cos(\bar{\omega}t - \bar{k}z + \delta_y(t))$$

- $\bar{\omega} \rightarrow$  Mean frequency
- $\bar{k} \rightarrow$  Mean wave vector
- $a_x(t), a_y(t), \delta_x(t), \delta_y(t) \rightarrow$  Slowly varying in comparison with the periodic term  $\exp(j\bar{\omega}t)$

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- The receiving antenna measures the target scattered narrowband wave during an interval of time  $T$  (Azimuth integration time)
- If  $T < \tau$  (Target coherence time)  
 $\Rightarrow a_x(t), a_y(t), \delta_x(t), \text{ and } \delta_y(t) \rightarrow \text{Assumed constant}$
- The wave then behaves in the time interval  $T$  like a monochromatic wave with mean frequency  $\bar{\omega}$
- *The Jones vector or the Stokes vector can be used to characterize the polarization of the monochromatic wave that is said to be a completely polarized wave*

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- For a longer time interval  $T$  the Electric field components  $a_x, a_y$  and the phases  $\delta_x, \delta_y$  are time varying

- → Wave is *partially polarized*

- In this case the parameters that characterizes the polarization wave should be time averaged



*Under the condition of signal wide - sense stationarity and ergodic conditions*

?

- The correlation between the time Electric field components is necessary to characterize partially polarized waves → *Coherency matrix* measurement

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- The coherency matrix is an interesting tool that permits observable parameters of a partially polarized wave to be measured
  - To deal with observables quantities, 2 quadratic forms of the quadratic products of  $\vec{E}$  and  $\vec{E}^{*T}$  are considered
    - Time averaged *total intensity*  $\rightarrow \langle \vec{E}^{*T} \cdot \vec{E} \rangle$
    - *Coherency (2x2) Hermitian matrix*  $\rightarrow [J] = \langle \vec{E} \cdot \vec{E}^{*T} \rangle$
- $\langle \dots \rangle \rightarrow$  Ensemble average ?

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- **$[J]$  is Hermitian positive semi-definite**
  - **Real non-negative eigenvalues**
- **The fact that  $[J] = [J]^*{}^T$  makes  $[J]$  an observable quantity**
- **The trace  $s_0$  of the matrix  $[J]$  is the total intensity of the wave**

$$s_0 = \|\vec{E}\|^2 = \text{trace}[J]$$

- ***The coherency matrix  $[J]$  is equivalent to the density matrix of Von Neuman that is widely used in Quantum Mechanics***

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- The *four* elements of the coherency matrix  $[J]$  are uniquely associated with the wave
- The unique set is intimately related to the appropriate degree of the Electric fields in the two orthogonal direction

$$\begin{aligned} [J] = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} &= \begin{bmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{bmatrix} \\ &= \begin{bmatrix} \langle |a_x|^2 \rangle & \langle a_x a_y \cdot \exp(-j\delta) \rangle \\ \langle a_x a_y \cdot \exp(j\delta) \rangle & \langle |a_y|^2 \rangle \end{bmatrix} \end{aligned}$$

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- If the axes  $(x, y)$  are rotated about the direction of propagation  $\rightarrow$  the coherency matrix changes
- However, the determinant  $|J|$  of  $[J]$ , the two real non-negative eigenvalues  $\lambda_1$  and  $\lambda_2$ , as well as the trace of the Hermitian coherency matrix  $[J]$  remains **rotation invariant**
- *Combination of these entities leads to rotation invariant parameter of the wave  $\rightarrow$  **Degree of Polarization (DOP)***

$$p = \sqrt{1 - \frac{4 \cdot |J|}{\text{Tr}(J)^2}} = \frac{|\lambda_2 - \lambda_1|}{\lambda_2 + \lambda_1}$$

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- **The rotation invariant parameter DOP has a physical significance**
  - **The wave is considered to be completely polarized if  $|J| = 0 \Rightarrow DOP = 1$**
  - **The wave is said to be completely unpolarized if the intensity of its components in any direction perpendicular to the direction of propagation is a constant**  
 **$\Rightarrow |J|$  is diagonal and the elements are identical  $\Rightarrow DOP = 0$**

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- A partially polarized wave can also be related to the elements of the coherency matrix

$$s_0 = J_{xx} + J_{yy} = \langle |a_x|^2 \rangle + \langle |a_y|^2 \rangle$$

$$s_1 = J_{xx} - J_{yy} = \langle |a_x|^2 \rangle - \langle |a_y|^2 \rangle$$

$$s_2 = J_{xy} + J_{yx} = 2 \langle a_x a_y \cos(\delta) \rangle$$

$$s_3 = j \cdot (J_{xy} - J_{yx}) = 2 \langle a_x a_y \sin(\delta) \rangle$$

- There is a one-to-one correspondence between the coherency matrix and the Stokes vector

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- Radar energy backscattered from landscape will often be polarized
- If the scattering is from random scattering media or time-varying scatterers the wave will be either partially polarized or completely unpolarized
- For a totally unpolarized wave the two amplitudes fluctuates randomly without any relationship between them → *Amplitude variation is uncorrelated*
- *The relative phase between the components would be totally random*

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

▪ If the two orthogonal components  $\rightarrow$  (say H and V) are totally random such that there is no preferred polarization

$$\Rightarrow \langle a_H^2 \rangle = \langle a_V^2 \rangle \Rightarrow s_0 = 2\langle a_H^2 \rangle$$

$$s_1 = 0$$

$$s_2 = s_3 = 0$$

*(The time average of the trigonometric function of a randomly varying angle will be zero )*

$$\mathbf{s}^{unpol} = \begin{bmatrix} 2\langle a_H^2 \rangle \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- **Partially polarized wave results from the addition of unpolarized and polarized component**

$$\mathbf{s}^{pol} = \begin{bmatrix} s_0^{pol} \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \quad \text{and} \quad \mathbf{s}^{unpol} = \begin{bmatrix} s_0^{unpol} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\mathbf{s}^{Total} = \begin{bmatrix} s_0^{pol} + s_0^{unpol} \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} s_0^{Total} \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

$$s_0^{pol} = \sqrt{s_1^2 + s_2^2 + s_3^2}$$

# Polarization (Stokes Vector)

Quasi-monochromatic and Partially polarized waves

- **Degree of polarization (DOP)**

$$DOP = \frac{\text{Power density of the polarized part}}{\text{Total power density}}$$

$$DOP = \frac{s_0^{pol}}{s_0^{pol} + s_0^{unpol}} = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2}}{s_0^{Total}}$$

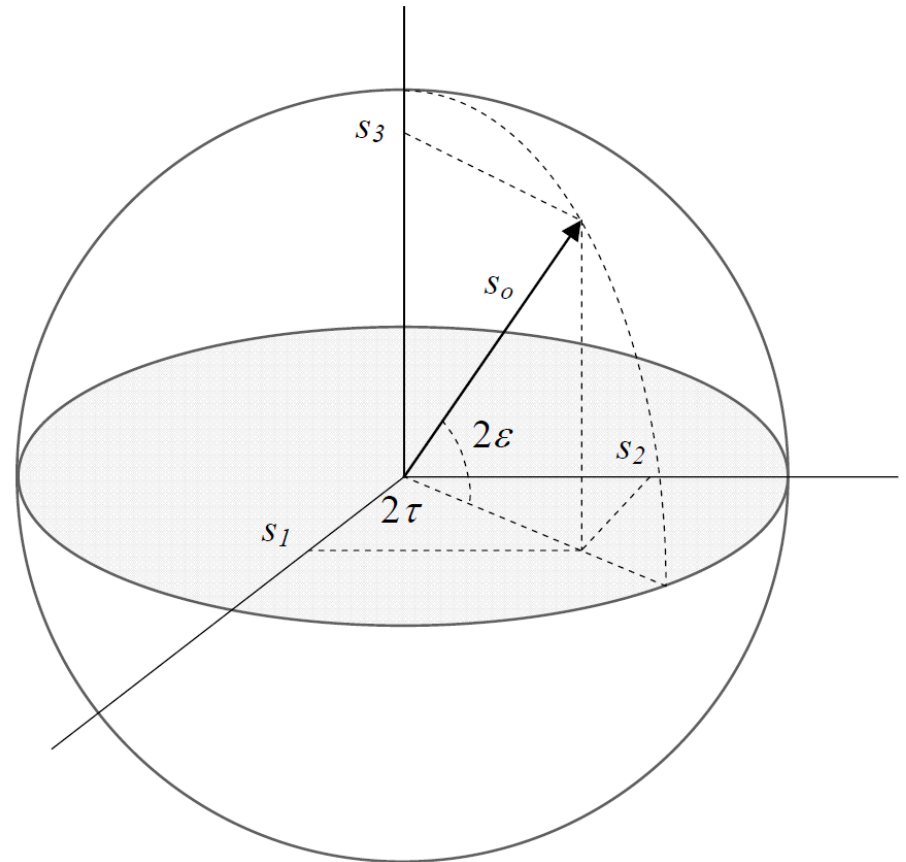
# Polarization (Stokes Vector)

## Poincare Sphere Representation

- A very interesting geometric representation of the Stokes parameters and the state of polarization of a wave emerges from

$$s_0^2 = s_1^2 + s_2^2 + s_3^2$$

- Equation of a sphere in the  $s_1, s_2, s_3$  coordinate space



# Polarization (Stokes Vector)

## Poincare Sphere Representation

- The sphere has radius of  $S_0$  and its surface is the locus of all possible polarization states.
- The polarization of a wave can be described by the amplitudes of its two orthogonal components  $a_H$  and  $a_V$  and their relative phase  $\delta$
- The polarization of a wave can alternatively be also described by the angles of the polarization ellipse  $\varepsilon$  and  $\tau$  and the wave intensity  $S_0$

# Polarization (Stokes Vector)

## Poincare Sphere Representation

$$\sin 2\varepsilon = \frac{2a_H a_V}{a_H^2 + a_V^2} \sin \delta = \frac{s_3}{s_0} \text{ thus } s_3 = s_0 \sin 2\varepsilon$$

$$\tan 2\tau = \frac{2a_H a_V}{a_H^2 - a_V^2} \cos \delta = \frac{s_2}{s_1} \text{ thus } s_2 = s_1 \tan 2\tau$$

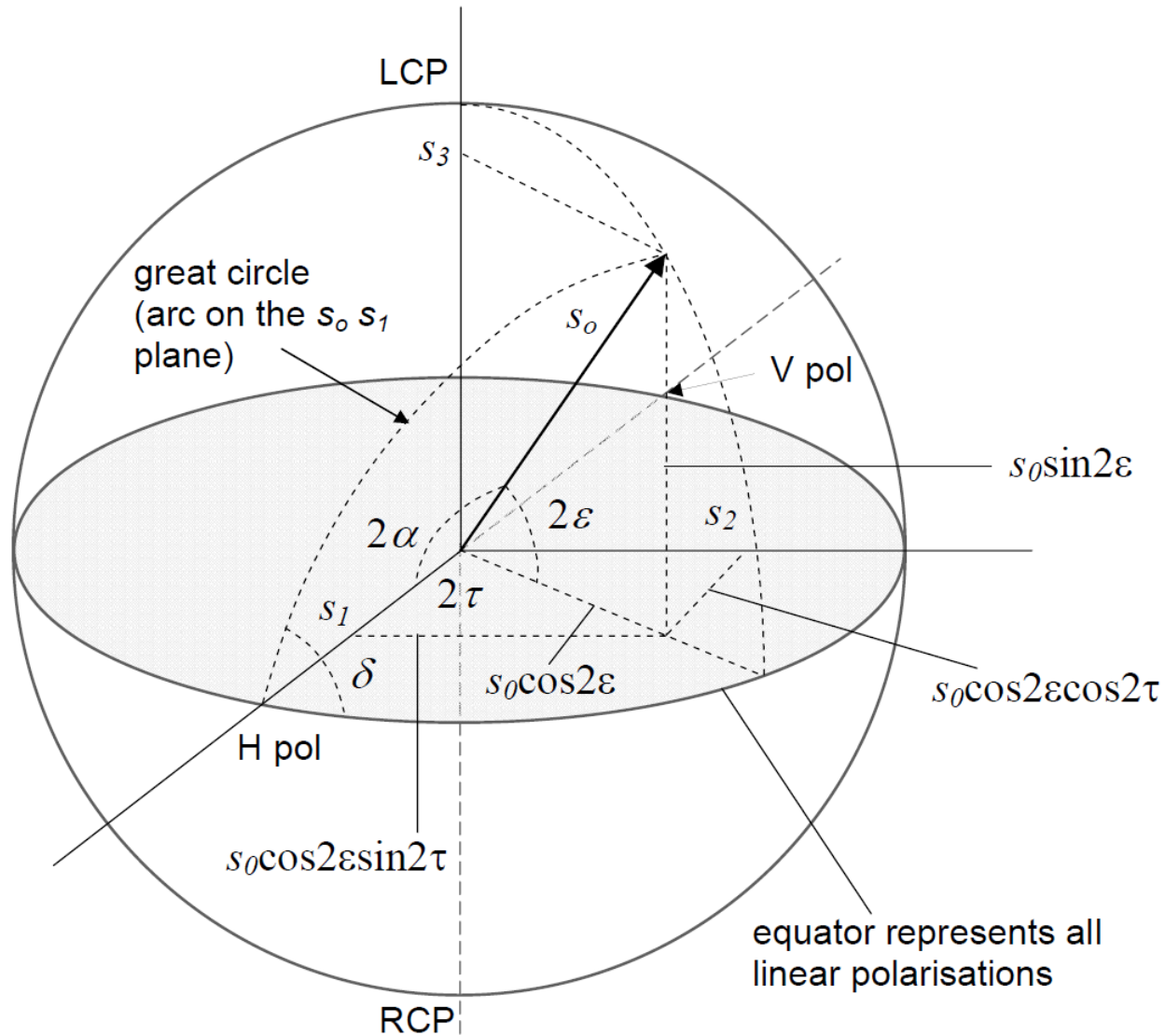
$$s_0^2 = s_1^2 + s_1^2 \tan^2 2\tau + s_0^2 \sin^2 2\varepsilon$$

$$s_1 = s_0 \cos 2\varepsilon \cos 2\tau$$

$$s_2 = s_0 \cos 2\varepsilon \sin 2\tau$$

# Polarization (Stokes Vector)

## Poincare Sphere Representation



# Polarization (Stokes Vector)

## Poincare Sphere Representation

- For partially polarized wave the point lies inside the sphere

$$s_1^2 + s_2^2 + s_3^2 < s_0^2$$

- The origin represents the case of unpolarized radiation