

Plane Waves and Polarization

The simplest EM waves are uniform plane waves propagating in some fixed direction, say the z-direction, in a lossless medium $\{\epsilon, \mu\}$.

The assumption means that the field has no dependence on the transverse coordinates x,y and are function only of z, t. Thus we look for solution of Maxwell Equations for:

$$\vec{E}(x, y, z, t) = \vec{E}(z, t) \quad \text{and}$$

$$\vec{H}(x, y, z, t) = \vec{H}(z, t)$$

$$\vec{E}(z, t) = \hat{x} E_x(z, t) + \hat{y} E_y(z, t)$$

$$\vec{H}(z, t) = \hat{x} H_x(z, t) + \hat{y} H_y(z, t)$$

Monochromatic Waves:

Uniform, single frequency plane wave propagating in lossless medium is obtained as a special case by assuming harmonic time dependency.

$$\vec{E}(x, y, z, t) = \vec{E}(z) e^{i\omega t}$$

$$\vec{H}(x, y, z, t) = \vec{H}(z) e^{i\omega t}$$

Polarization:

Consider a forward moving wave and let

$$\vec{E}_0 = \hat{x} A + \hat{y} B$$

be its complex-valued phasor amplitude so that

$$\vec{E}(z) = \vec{E}_0 e^{-jkz} = (\hat{x} A + \hat{y} B) e^{-ikz}$$

The time-varying field is:

$$\vec{E}(z, t) = (\hat{x} A + \hat{y} B) e^{i\omega t - ikz}$$

The polarization of a plane wave is defined to be the direction of the electric field vector.

More precisely, polarization is the direction of the time-varying real value of the field.

$$\vec{E}(z, t) = \text{Re}\{\vec{E}(z, t)\}$$

At any fixed point z, the vector $\vec{E}(z, t)$ may be along a fixed linear direction or it may be rotating as a function of time, along a circle or an ellipse.

The polarization properties of the plane wave are determined by the relative magnitude and phases of complex-valued constants A, B.

Writing them in their polar forms:

$$A = A_+ e^{i\varphi_a}$$

$$B = B_+ e^{i\varphi_b}$$

A_+ and B_+ are positive magnitudes.

$$\begin{aligned}\vec{E}(z, t) &= (\hat{x} A_+ e^{i\varphi_a} + \hat{y} B_+ e^{i\varphi_b}) e^{i\omega t - ikz} \\ &= \hat{x} A_+ e^{(\omega t - kz + \varphi_a)} + \hat{y} B_+ e^{(\omega t - kz + \varphi_b)}\end{aligned}$$

Extracting the real parts:

$$\begin{aligned}\vec{E}(z, t) &= \text{Re}\{\vec{E}(z, t)\} \\ &= \hat{x} E_x(z, t) + \hat{y} E_y(z, t)\end{aligned}$$

we find

$$E_x(z, t) = A \cos(\omega t - kz + \varphi_a)$$

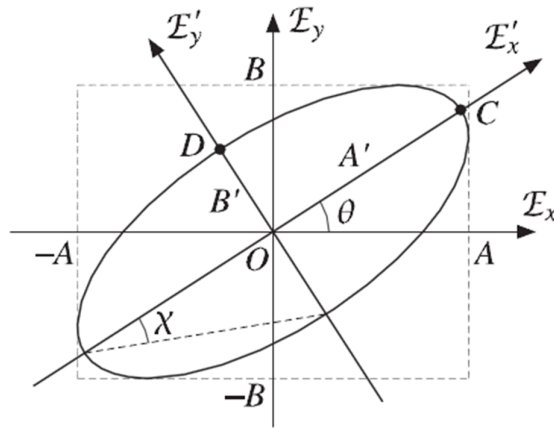
$$E_y(z, t) = B \cos(\omega t - kz + \varphi_b)$$

To determine the polarization of the wave, we consider the time-dependence of these fields at some fixed point along z axis, say $z=0$.

$$E_x(t) = A \cos(\omega t + \varphi_a)$$

$$E_y(t) = B \cos(\omega t + \varphi_b)$$

We denote $\varphi = \varphi_a - \varphi_b$ as the relative phase.



The tilt angle θ is given by

$$\tan 2\theta = \frac{2AB}{A^2 - B^2} \cos \varphi$$

The ellipse semi axis A' and B' are given by

$$A' = \sqrt{\frac{1}{2}(A^2 + B^2) + \frac{s}{2}\sqrt{(A^2 - B^2)^2 + 4A^2B^2 \cos^2 \varphi}}$$

$$B' = \sqrt{\frac{1}{2}(A^2 + B^2) - \frac{s}{2}\sqrt{(A^2 - B^2)^2 + 4A^2B^2 \cos^2 \varphi}}$$

$$s = \text{sign}(A - B)$$

$$E'_x = E_x \cos \theta + E_y \sin \theta$$

$$E'_y = E_y \cos \theta - E_x \sin \theta$$

$$\sin 2\aleph = \frac{2AB}{A^2 + B^2} |\sin \varphi|$$

$$-\frac{\pi}{4} \leq \aleph \leq \frac{\pi}{4}$$

It can be shown that

$$\tan \aleph = B'/A' \text{ or } A'/B'$$

whichever is less than one.

Problem:

Determine the real value of electric and magnetic field components and the polarisation of the following fields specified in the phasor forms:

- $\vec{E}(z) = -3j \hat{x} e^{-jkz}$
- $\vec{E}(z) = (3 \hat{x} + 4 \hat{y}) e^{+jkz}$
- $\vec{E}(z) = (-4 \hat{x} + 3 \hat{y}) e^{-jkz}$
- $\vec{E}(z) = (3e^{j\pi/3} \hat{x} + 3 \hat{y}) e^{+jkz}$
- $\vec{E}(z) = (4 \hat{x} + 3e^{-j\pi/4} \hat{y}) e^{-jkz}$
- $\vec{E}(z) = (3e^{-j\pi/8} \hat{x} + 4e^{j\pi/8} \hat{y}) e^{+jkz}$
- $\vec{E}(z) = (4e^{j\pi/9} \hat{x} + 3e^{-j\pi/2} \hat{y}) e^{-jkz}$
- $\vec{E}(z) = (3e^{-j\pi/2} \hat{x} + 4e^{j\pi/4} \hat{y}) e^{+jkz}$