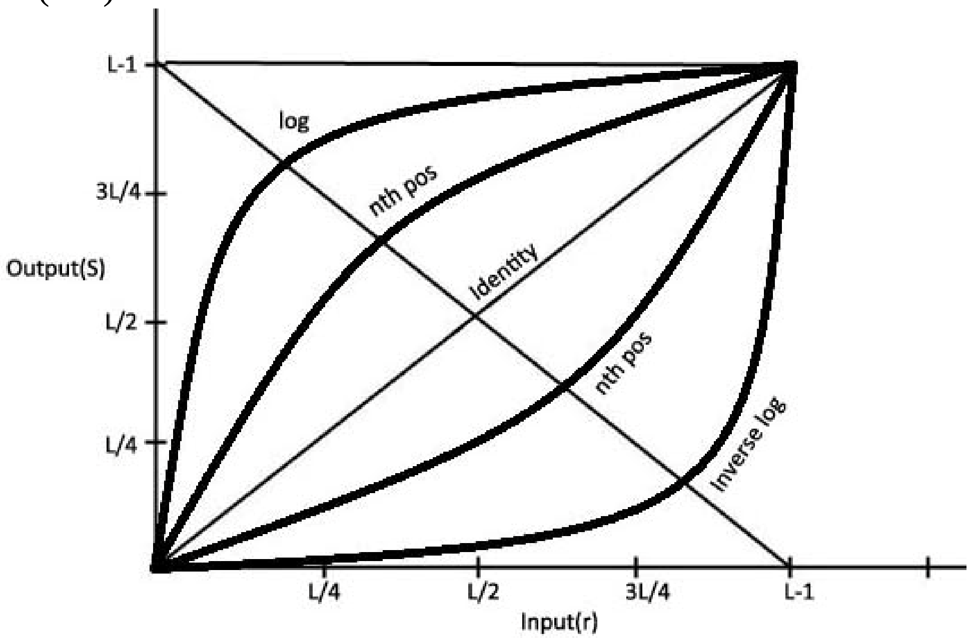
S=T(r)

1) Negative images

Intensity level [0,L-1]

S=(L-1)-r



2) Log Transform

S=c log(1+r)

C →constant, r≥ 0

Maps a narrow range of low intensity values in the input, into a wider range of output levels. Opposite is true for high value of input levels. Transformation of this type to expand the dark values pixel in an image. While compressing the higher values. The opposite is true for inverse-log.

The power law transformation are much more suitablefor the purpose. The log function has the important characteristic that it comprises the dynamic range of images with large variation in pixel values.

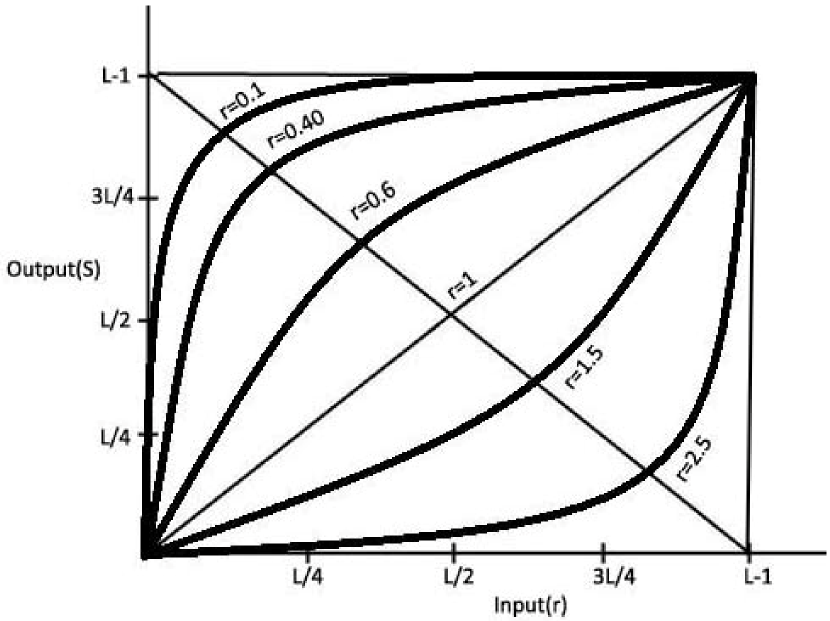
(Example → Fourier Transform → 0-10^6 values)

3) Power law (Gamma)

$S = cr^{\gamma}$, c,γ≥ 0

Sometimes written as

$S = c(r + \varepsilon)^{\gamma}$



Fractional power (γ) map a narrow range of dark input values into a wider range of output values, with the opposite being true for higher values. However we notice, unlike the log transform, we have a family of possible transform curves obtained by varying γ.
It is Referred to as gamma law.

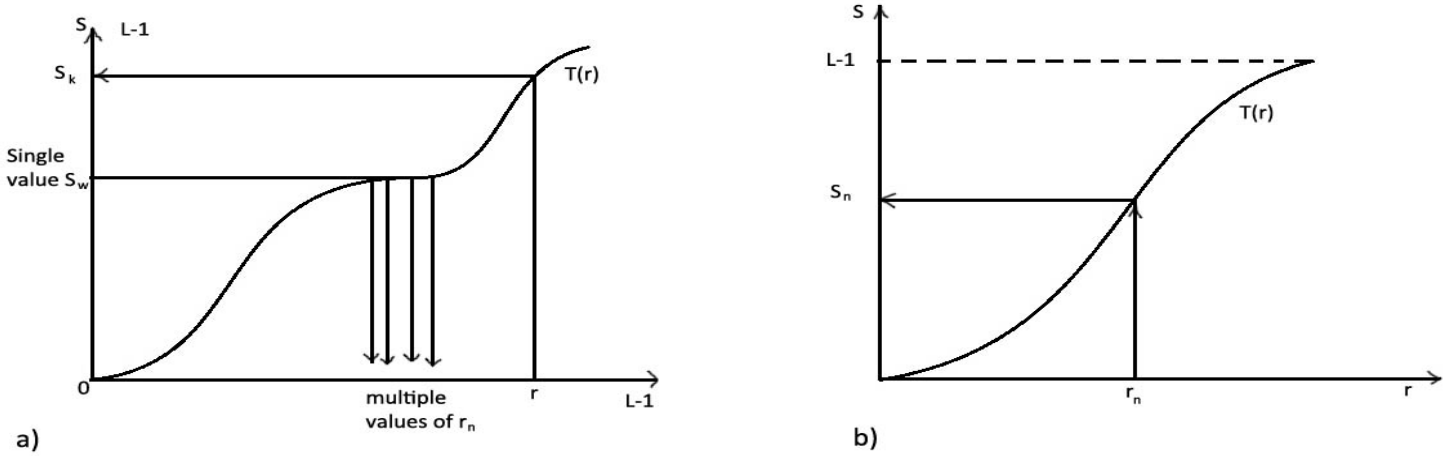
Exponential stretching is the opposite of a logarithmic law.

Histogram processing-
The histogram of a digital image with intensity level [0 , L-1] is a discrete function $h(r_k)=n_k$, where r_k is the kth intensity value and n_k *is the number* of pixels in the image with intensity r_k.
normalize histogram, $p(r_k)=n_k/MN$

Histogram Equalization-
Consider for a normal continuous intensity values and let the variable r denote intensities of an image [0,L-1] with
r=0 (blank) and r=L-1 (white)
$S=T(r) \quad , 0 \leq r \leq L-1$

We assume that
a) T(r) is monotonically increasing function in the interval $0 \leq r \leq L-1$ and
b) $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$

The requirement of condition a) that T(r) is monotonically increasing guarantees that the output intensity values will never be less than corresponding input values.



a)rectifies condition a) and b) perfectly, aim for mapping r→s, inverse could be a problem.

If $p_r(r)$ and $p_s(s)$ denote the PDF of r and s, and if $p_r(r)$ and T(r) are known and continuous and differentiable over the range values, then

$ps(s) = pr(r).\left|\frac{dr}{ds}\right|$

A transformation of particular importance in image processing has the form

$S=T(r) = (L-1)\int_0^r p_r(\omega)d\omega$

The RHS in the equation is recognized as the CDF of random variable r.

where r=L-1 the upper limit, the integral is 1.

We know from Leibnitz rule that the derivative of a definite integral with respect to its upper limit is the integration evaluated of the limit.

$$\frac{dS}{dr} = \frac{dT(r)}{dr}$$

$$=(L-1).\frac{d}{dr}\int_0^r p_r(\omega)d\omega$$

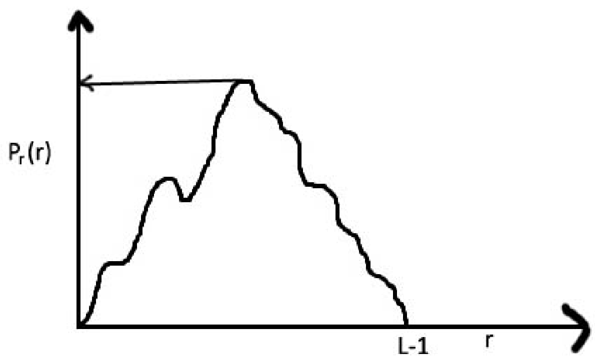
$$=(L-1).p_r(r)$$

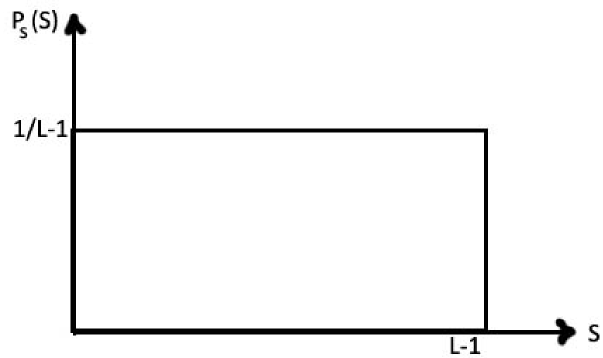
$$P_s(s)=p_r(r).\left|\frac{dr}{ds}\right|$$

$$=Pr(r).\left|\frac{1}{(L-1)p_r(r)}\right|$$

$$=\frac{1}{(L-1)},0\leq s\leq L-1$$

It is important to note that T(r) depend on $p_r(r)$ but $p_s(s)$ is always uniform, independent of $p_r(r)$.





<u>Example</u>

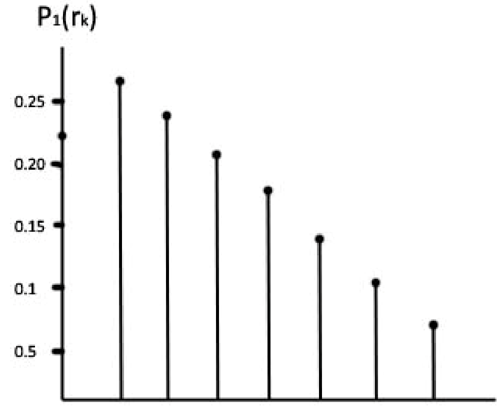
We have a 3 bit image (L=8) of size 64x64 (MN=4096),
Intensity levels are [0,L-1]=[0,7]

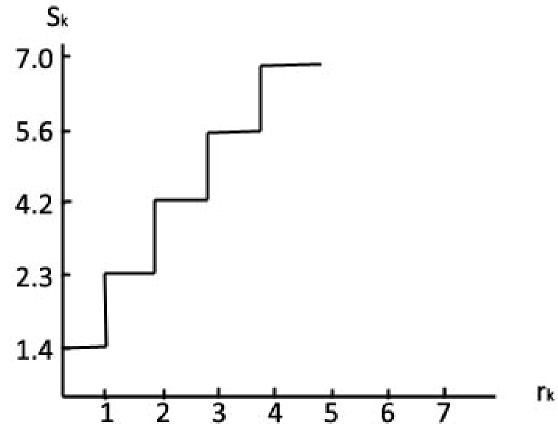
R_k	N_k	$P_r(n_k)=n_k/MN$
$R_0=0$	790	0.19
$R_1=1$	1023	0.25
$R_2=2$	850	0.21
$R_3=3$	656	0.16
$R_4=4$	329	0.05
$R_5=5$	245	0.06
$R_6=6$	122	0.03
$R_7=7$	81	0.02

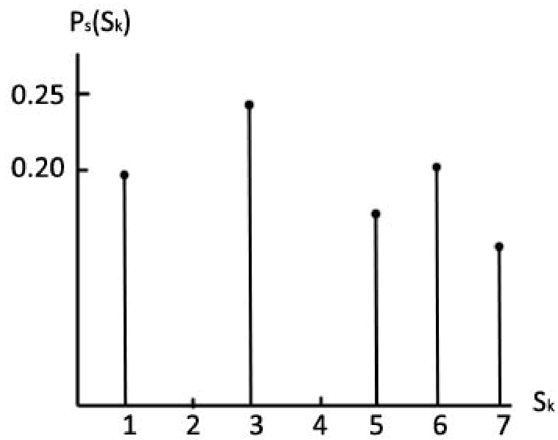
$S_o = T(r_o) = 7\sum_{j=0}^{0} P_r(r_j) = 7\text{x}P_r(r_0) = 1.33$

$S_1 = T(r_1) = 7\sum_{j=0}^{1} P_r(r_j) = 7\text{x } P_r(r_0) + 7\text{x } P_r(r_1) = 3.08$

S_2=4.55, S_3=5.67, S_4=6.23, S_5=6.56, S_6=6.86, S_7=7.00







S_o= 1.33→1
S_1= 3.08→3
S_2=4.55→5
S_3=5.67→6
S_4=6.23→6
S_5=6.56→7
S_6=6.86→7
S_7=7.00→7

There are only five distinct intensity levels. Histogram is an approximation of a pdf and perfectly flat histograms are rare in applications.